**Math 231 – HW 7.5 Name: Troy Jeffery**

***Remember -- FORMAT is as important as CONTENT – get them both right!***

3.3 11 3.6 10, 23, 6.

***Warm-up Questions -- Negations of Quantified Statements.***

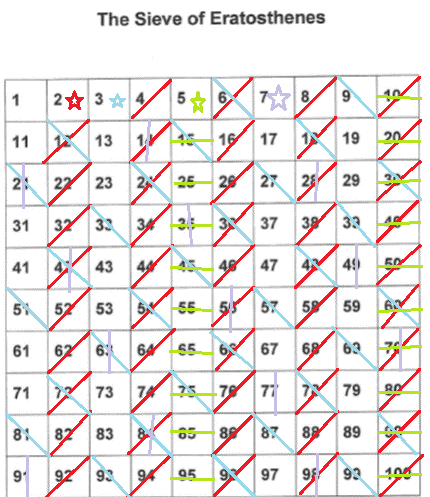
|  |  |
| --- | --- |
| *The original quantified statement, in words:*  Any rational number is also a real number. | *The negation of the original statement, in words:*  There is a rational number that is not a real number. |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

|  |  |
| --- | --- |
| *The original quantified statement, in words:*  There is a rational number that is also an integer. | *The negation of the original statement, in words:*  No rational number is also an integer. |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

**3.3 (11)** Do a formal proof of the theorem:

|  |  |
| --- | --- |
| Theorem: | If n = 4k+1 (for some integer k), then n2-1 is divisible by 8. |
| Proof: | Since n = 4k+1 for some integer k  Then,        The sum or product of an integer is an integer. |

**3.3 (29)** Use the sieve of Eratosthenes to find all the prime numbers less than 100. (read the rest of the problem in the book!)



Here's the list of prime numbers less than 100:

1,2,3,5,7, 11,13,17,19,23, 29,31,37,41,43, 47,53,59,61,67, 71,73,79,83,89, 97

**3.6 (10)** Prove the theorem in two ways -- by contraposition and by contradiction.

Theorem: If the square of an integer is odd, then the original integer is odd.

*By contraposition:*

Exploration: Rewrite the theorem as an if-then conditional:

∀ n∈, if ,then .

Write the contraposition of your if-then statement:

∀ n∈, if , then .

*Now, let's do the proof by contraposition:*

|  |  |
| --- | --- |
| Theorem: | If the square of an integer is odd, then the original integer is odd. |
| Proof: | It is sufficient to show that: ∀ n∈, if , then .  Since we know that n is not odd, we can assume that for some integer k.  So,        Therefore, we know that both the number and its square are not odd numbers. |

*By contradiction:*

Exploration: Write the negation of the theorem:

There exists an integer’s square that is even although it’s base is odd.

*Now, let's do the proof by contradiction:*

|  |  |
| --- | --- |
| Theorem: | If the square of an integer is odd, then the original integer is odd. |
| Proof: | Suppose not. In other words, there must be some integer whose square is even, but base is odd.  Since we know that the square is even we can assume that  Because the product or sum of integers are integers.  We find that |

**3.6 (23)** Prove the theorem in two ways -- by contraposition and by contradiction.

Theorem: If r is any non-zero rational number, and s is any irrational number, then  is irrational.

*By contraposition:*

Exploration: Rewrite the theorem as an if-then conditional:

Write the contraposition of your if-then statement:

*Now, let's do the proof by contraposition:*

|  |  |
| --- | --- |
| Theorem: | If r is any non-zero rational number, and s is any irrational number, then  is irrational. |
| Proof: | It is sufficient to show that:  Since r and is a non-zero rational, we can assume that      So,  Because the sum or product of integers is an integer. |

*By contradiction:*

Exploration: Write the negation of the theorem:

*Now, let's do the proof by contradiction:*

|  |  |
| --- | --- |
| Theorem: | If r is any non-zero rational number, and s is any irrational number, then  is irrational. |
| Proof: | Suppose not. In other words, there exists a non-zero rational number such that if s is irrational, then is rational.  Since we know that both r and r/s are non-zero rationals we can assume:    So,  Because the sum or product of integers is an integer.      We’ve proved that s is a rational number, but we assumed it would be irrational.  Therefore, |

**3.6 (6)** Finish the following proof.

|  |
| --- |
| *Extra question: Rewrite the above statement using symbols instead of words.* |

|  |  |
| --- | --- |
| Theorem: | The difference of any rational and any irrational number is irrational. |
| Proof: | Suppose not. In other words, suppose that there is a rational number x and an irrational number y such that  is rational.  Since x is rational,  Since z is rational, .  Then,        Because the sum or product of integers is an integer.      Because we assumed that y was irrational, |